Solutions 9; Phys 191 Name ______________

1. (40 points) You have two events (ct_1, x_1) and (ct_2, x_2). In the frame of reference where $x_1 = x_2$, call the time separating these events $\Delta t_p = t_2 - t_1$. (Δt_p is the "proper time" interval.) You then measure the time interval $\Delta t' = t'_2 - t'_1$ in another frame of reference moving at $v = \beta c$ relative to the proper time frame. Find the relationship between Δt_p and $\Delta t'$.

Answer: Doing the Lorentz transformation,

$$ct'_2 - ct'_1 = (\gamma ct_2 - \beta\gamma x_2) - (\gamma ct_1 - \beta\gamma x_1) = \gamma(ct_2 - ct_1) \quad \Rightarrow \quad \Delta t' = \gamma \Delta t_p$$

This is time dilation: Since $\gamma \geq 1$, $\Delta t' \geq \Delta t_p$. The proper time is the minimum time interval between events, and time intervals will expand in non-proper time frames.

2. (60 points) You measure the length of an object in a frame where it is at rest: Get the location of one end at (ct_1, x_1) and the other end at (ct_2, x_2). Since the object is at rest, what ct_1 and ct_2 are doesn't matter. The "proper length" $\Delta x_p = x_2 - x_1$ will not change, since x_2 and x_1 do not change. Then, an observer measures the length $\Delta x' = x'_2 - x'_1$ in another frame of reference moving at $v = \beta c$ relative to the proper length frame. However, since the object is moving in that frame, they have to obtain (ct'_1, x'_1) and (ct'_2, x'_2) while making sure that $t'_1 = t'_2$. Find the relationship between Δx_p and $\Delta x'$.

Answer: Since we know that $ct'_1 = ct'_2$ in the frame where the object is moving, we need to Lorentz transform from that frame to the proper length frame. Technically, it's an inverse Lorentz transform, since the proper length frame is moving at $v = -\beta c$ with respect to the moving frame, but that won't matter for the result.

$$x_2 - x_1 = (\gamma x'_2 + \beta\gamma ct'_2) - (\gamma x'_1 + \beta\gamma ct'_1) = \gamma(x'_2 - x'_1) \quad \Rightarrow \quad \Delta x' = \frac{1}{\gamma}\Delta x_p$$

Since $\gamma \geq 1$, $\Delta x' \leq \Delta x_p$. Lengths contract. The proper length is a maximum, and an object's length will always be shorter in frames where it is moving.