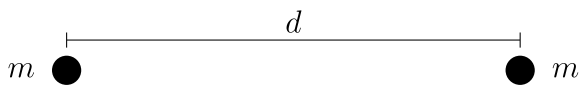
Solutions to Exam 2; Phys 185

1. (20 points) You have two identical stars with mass m in outer space, separated by a distance d, in a circular orbit around their common center of mass. The only forces on them are their mutual gravitational attraction. Find the period T of their orbit.

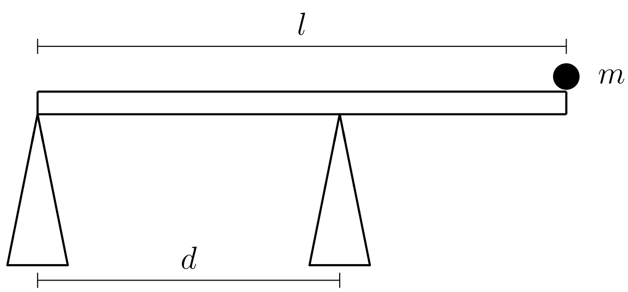


Answer: The center of mass is exactly in the middle, so the radius of the orbit of a star is $d/2$. Setting up $\sum \vec{F} = m\vec{a}$ for one of the stars results in:

$$G\frac{m^2}{d^2} = m\frac{v^2}{(d/2)} = m\frac{\left(\frac{2\pi(d/2)}{T}\right)^2}{(d/2)} \quad \Rightarrow \quad T = \pi\sqrt{\frac{2d^3}{Gm}}$$

2. (30 points) You have a uniform density rectangular plank with mass M and length l, resting with one end on a support and its middle part on another support a distance d from the first support. At the end of the plank, there is a mass m, where the size of this mass is much smaller than l. Find an equation for the minimum distance d where this whole arrangement remains stable, without tipping over.

Hint: Call the normal force applied by the first support $\vec{n}_1$. What must n_1 be at the minimum d, when the whole thing is just about to tip over?



Answer: The center of mass of the plank is at its middle, a distance of $l/2$ from the left end.

If nothing is moving, two equations must be satisfied: $\sum F_y = 0$ and $\sum \tau = 0$. Take the axis of rotation to be at the left end at the plank. (It doesn't matter where you choose the axis to be.) The normal force applied by the small mass on the plank is downward and is mg in magnitude.

$$n_1 + n_2 = Mg + mg \qquad \text{and} \qquad n_2 d = Mg(l/2) + mgl$$

If the whole thing is just about to tip over, $n_1 = 0$. (Note that $\vec{n}_1$ cannot be directed downward.) Therefore, solving for n_2 from the force equation a d substituting it in the torque equation,

$$(M+m)gd = \frac{Mgl}{2} + mgl \qquad \Rightarrow \qquad d = \left(\frac{M/2+m}{M+m}\right) l$$

3. (50 points) You perform a collision experiment much like what you did in the lab, using expensive equipment that makes friction and drag negligible and keeps the tracks almost perfectly horizontal. The first cart has mass 0.500 kg, and it travels toward the second with an initial velocity of $v_{1i} = 1.000\,\text{m/s}$. The second cart has mass 1.500 kg and it starts at rest. During the collision, however, you lose a fraction r of the initial total kinetic energy of the system. (If, for example, $r = 0.1$, you lose 10% of your starting energy to sound and heat produced during the collision.)

(a) Find equations for the post-collision velocities of the carts, v_{1f} and v_{2f}. Your results will depend on r. You will need the quadratic formula: if $ax^2 + bx + c = 0$, the two solutions are $x_{1,2} = (-b \pm \sqrt{b^2 - 4ac})/(2a)$. After the collision, you must have $v_{1f} \leq v_{2f}$, since the carts cannot pass through one another. Choose the solution with the + or the − sign accordingly.

Answer: Momentum conservation gives

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad \Rightarrow \qquad v_{2f} = \frac{m_1}{m_2}(v_{1i} - v_{1f}) = \frac{1}{3}(1 - v_{1f})$$

The energy loss is

$$E_{\text{loss}} = r\left(\tfrac{1}{2} m_1 v_{1i}^2\right)$$

Energy conservation:

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_1 v_{2f}^2 + E_{\text{loss}}$$

Putting in the loss, the momentum conservation result, and the numbers, we get

$$(1 - r)\left(\tfrac{1}{2}(0.5)(1)^2\right) = \tfrac{1}{2}(0.5) v_{1f}^2 + \tfrac{1}{2}(1.5)\frac{1}{9}(1 - v_{1f})^2$$

Using $(1 - v_{1f})^2 = 1 - 2v_{1f} + v_{1f}^2$, we can arrange this into a quadratic equation:

$$\tfrac{1}{3} v_{1f}^2 - \tfrac{1}{6} v + \tfrac{1}{4}(r - \tfrac{2}{3}) = 0$$

The solutions are:

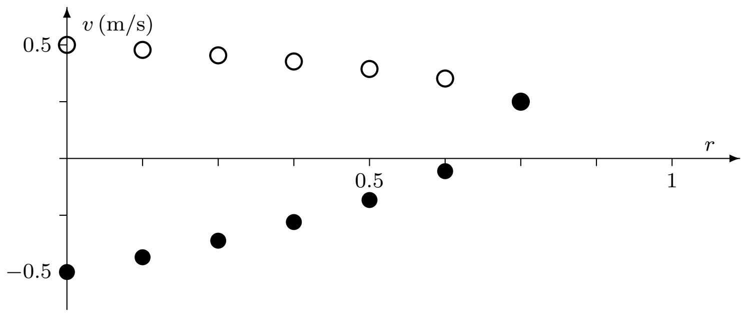
$$v_{1f} = \frac{3}{2}\left(\frac{1}{6} \pm \sqrt{\frac{1}{36} - \frac{r - 2/3}{3}}\right) = \frac{3}{2}\left(\frac{1}{6} \pm \sqrt{\frac{9}{36} - \frac{r}{3}}\right)$$

Since $v_{1f} \leq v_{2f}$, we should keep the smaller result, which is the one with the − sign.

(b) Fill in the following table. First obtain values for v_{1f} and v_{2f} for $r = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$. Do the others only if you have time.

r	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
v_{1f} (m/s)	-0.500	-0.435	-0.362	-0.280	-0.183	-0.056	0.250		
v_{2f} (m/s)	0.500	0.478	0.454	0.427	0.394	0.352	0.250		

(c) Sketch graphs of v_{1f} and v_{2f}.



(d) If you get any notable or strange results from your calculations, interpret them: tell me what they mean—what must be going on?

Answer: The maximum energy loss comes when the carts stick together: notice that the final velocities are the same at $r = \frac{3}{4}$. But for $r > \frac{3}{4}$, there cannot be such a collision, and you will end up trying to take the square root of a negative number.