Solutions to Exam 3; Phys 185

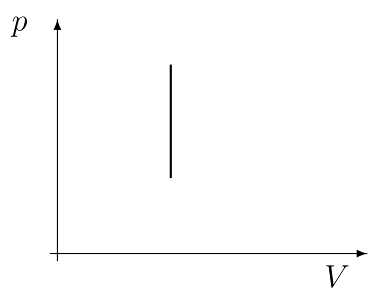
1. (10 points) List a few things you have recently learned about the nature of temperature. I'm not looking for a full definition of temperature—you don't have one. Just a list a few things (two or three will be enough) that have helped you improve your understanding of what temperature is. And don't write a long essay. You'll only need all this page if your writing is really large.

Answer: Your list will vary. I would hope it would include some of the following:

- Our everyday sense of hot and cold does not exactly translate to temperature. We might touch objects with the same temperature, and depending on their thermal conductivity, we may fell them to be hotter or colder compared to the other.
- Temperature is not a property of individual particles. It is a statistical quantity: it describes a large population of particles.
- For an ideal gas, temperature is proportional to the average kinetic energy of the molecules. But that is *only* for an ideal gas.

2. (40 points) In an ideal gas, the total thermal energy is $U = \frac{f}{2}NkT$. Here, f is a constant, and for a monatomic ideal gas, $f = 3$.

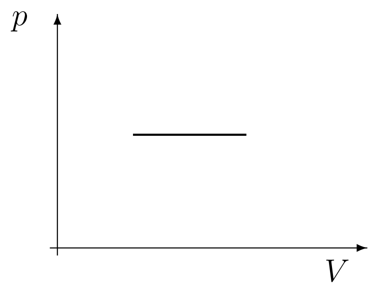
(a) Draw a p–V graph for an ideal gas that starts at an initial pressure p_i, and goes to a final p_f in a process where *the volume remains constant*. C_v, the heat capacity for an ideal gas at constant volume, is defined by $Q = C_v \Delta T$. Find C_v.



Answer: Due to energy conservation, $Q = \Delta U - W$. The area under the curve is zero, so $W = 0$. We have

$$Q = U_f - U_i = \frac{f}{2}NkT_f - \frac{f}{2}NkT_i = \frac{f}{2}Nk\Delta T \quad \Rightarrow \quad C_v = \frac{f}{2}Nk$$

(b) Draw a p–V graph for an ideal gas that starts at an initial volume V_i, and goes to a final V_f in a process where *the pressure remains constant*. C_p, the heat capacity for an ideal gas at constant pressure, is defined by $Q = C_p \Delta T$. Find C_p.



Answer: Now the area is $p(V_f - V_i)$, and therefore $W = -p(V_f - V_i)$.

$$Q = U_f - U_i + W = \frac{f}{2}Nk\Delta T + p(V_f - V_i) = \frac{f}{2}Nk\Delta T + Nk(T_f - T_i)$$

Therefore

$$C_p = \left(\frac{f}{2} + 1\right) Nk$$

3. (50 points) You have a person standing outside on a cold day, losing heat to her surroundings.

(a) She conducts heat through the soles of her shoes to the ground. The area of her shoe soles is $5.0 \times 10^{-2}\,\mathrm{m}^2$, the thickness of her soles is $1.5 \times 10^{-2}\,\mathrm{m}$, and the thermal conductivity of her soles is $k = 0.20\,\mathrm{W/m{\cdot}K}$. The temperature of the bottom of her feet is 35°C, and the outside temperature is 1°C. What is her heat loss rate due to conduction?

Answer:

$$\frac{dQ}{dt} = kA\frac{\Delta T}{L} = 23\,\mathrm{W}$$

(b) She will lose heat due to convection. In her conditions, the convective heat transfer coefficient $h = 5.0\,\mathrm{W/m^2{\cdot}K}$. Her total surface area exposed to the air is $2.0\,\mathrm{m}^2$. The temperature at the surface of her clothes is 31°C, and the outside temperature is 1°C. What is her heat loss rate due to convection?

Answer:

$$\frac{dQ}{dt} = hA\Delta T = 300\,\mathrm{W}$$

(c) She is wearing white clothes, with $e = 1$ in the infrared and $e = 0$ in visible wavelengths. That means she absorbs no radiation from the sun, but absorbs everything from the rest of her environment at 1°C. Her surface area is $2.0\,\mathrm{m}^2$. The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2{\cdot}K^4}$. What is her net heat loss due to radiation (emission minus absorption)?

Answer:

$$\frac{dQ}{dt} = e\sigma A(T_{\text{surf}}^4 - T_{\text{env}}^4) = 329\,\text{W}$$

(d) She then changes into black clothes and goes out again: she now has $e = 1$ for both infrared and visible radiation. *Everything* remains the same for her, except that now she does absorb sunlight. The surface temperature of the sun is 5800 K, but since the sun occupies only a tiny part of the sky, and some sunlight is absorbed by the atmosphere, you have to multiply your calculated absorption of sunlight (that assumes you're surrounded by the same temperature on all sides) by 3.7×10^{-6}. What is her net heat loss due to radiation (emission minus absorption)?

Answer:

$$\frac{dQ}{dt} = e\sigma A(T_{\text{surf}}^4 - T_{\text{env}}^4) - (3.7 \times 10^{-6})\sigma A T_{\text{sun}}^4 = (329 - 475)\,\text{W} = -146\,\text{W}$$

She actually gains heat!

(e) Does wearing black rather than white clothes make a significant difference? Make a *quantitative* argument.

Answer: With white clothes, the total heat loss rate was $23+300+329 = 652\,\text{W}$. With black clothes, that goes down to $23+300-146 = 177\,\text{W}$. That is a $(652-177)/652 = 73\%$ reduction in heat loss. It would make a big difference for anyone who was outside a long time.