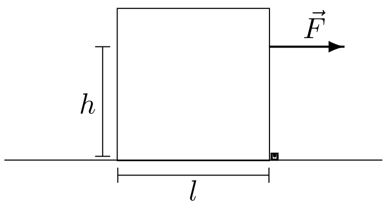
Solutions to Exam 4; Phys 185

1. (30 points) You are trying to move a box with mass m that has a length l on all sides, applying a horizontal force $\vec{F}$ at a height h above the floor. But there is a small obstacle at floor level that prevents movement. Find what F must be for the box to start to tip over (rotate around the obstacle).

Hint 1: The normal force from the obstacle has both a horizontal and a vertical component.

Hint 2: "Just starting to tip over" tells you something about one of the forces on the box. You need to figure out what this is.



Answer: When the box is just about to tip, the normal force from the floor $n_f = 0$. Call the components of the normal force from the obstacle n_{ox} and n_{oy}. The forces have to add to zero:

$$\sum F_x = F + n_{ox} = 0 \quad \Rightarrow \quad F = -n_{ox}$$

$$\sum F_y = n_f + n_{oy} - mg = 0 \quad \Rightarrow \quad n_{oy} = mg$$

The torque around the obstacle must also add up to zero:

$$\sum \tau = mg\frac{l}{2} - Fh = 0 \quad \Rightarrow \quad F = \frac{l}{2h}mg$$

And that's the answer.

2. (30 points) You're in a spaceship in a circular orbit around a planet. Your distance to the planet's center is r. But this is in a parallel universe where gravity is different. Gravity is still an attractive force along the line connecting centers of mass, but its distance-dependence is:

$$F_G = A\frac{m_1 m_2}{r^n}$$

where A is a universal constant, and the power $n \neq 2$.

You change your initial circular orbit by moving out to a distance of $2r$ to the planet's center. During your movement, the only forces applied on the spaceship are radial in direction, with zero tangential component. (Radial means straight inward or outward; tangential is tangent to a circle centered on the planet.) The maneuvers use a negligible amount of fuel, so that m_s, the mass of the ship, remains constant. And when you reach $2r$, the spaceship has a radial velocity component of zero, just as it was at r. And then you find that your spaceship continues at a perfect circular orbit with radius $2r$.

Find n.

Answer: When changing orbit, all the forces on the ship are radial: directed toward the axis of rotation. Therefore none of these forces produce any torque. Gravity is radial, which also does not produce torque. The angular momentum of the spaceship will be conserved. $L = I\omega$, and since the size of the starship is much smaller than r, we can approximate it as a point mass, with $I = m_s r^2$. Setting the initial and final angular momenta equal,

$$\left(m_s r^2\right)\omega_i = \left[m_s\,(2r)^2\right]\omega_f \qquad \Rightarrow \qquad \omega_f = \tfrac{1}{4}\omega_i$$

$\sum \vec{F} = m_s \vec{a}$ with the parallel universe gravity and uniform circular motion gives

$$A\frac{m_p m_s}{r^n} = m_s \omega_i^2 r \qquad \Rightarrow \qquad \omega_i = \sqrt{\frac{A m_p}{r^{n+1}}}$$

Now, for uniform circular motion with radius $2r$, the angular velocity must be

$$\omega_f = \sqrt{\frac{A m_p}{(2r)^{n+1}}} = \frac{1}{2^{(n+1)/2}}\sqrt{\frac{A m_p}{r^{n+1}}} = \frac{1}{2^{(n+1)/2}}\,\omega_i$$

Since the orbit remains a circle,

$$\frac{1}{4} = \frac{1}{2^2} = \frac{1}{2^{(n+1)/2}} \qquad \Rightarrow \qquad 2 = \frac{n+1}{2} \qquad \Rightarrow \qquad n = 3$$

3. (40 points) You drop one of your shoes, starting at rest, from a height of 0.40 m. It doesn't bounce, quickly settling on the soft, carpeted floor. Estimate by how much the temperature of the shoe rises *immediately after* colliding with the carpet? (It will cool down soon, but don't worry about that process.)

You will need to make some assumptions during your calculation. Be explicit about them all, and include a brief (one or two sentences) explanation for all your assumptions.

You will also need to make assumptions about some quantities you need—pick reasonable values. Explicitly indicate when you're choosing a value—I want to see where you do this.

There are also a few things for which you can't make reasonable assumptions, since you have too little experience with the relevant quantities. Should you need them, you can take the shoe's efficiency to be 5.0%, its coefficient of performance 3.2, specific heat capacity 1.0×10^3 J/kg·K, its thermal conductivity 0.25 W/m·K, convective heat transfer coefficient 5.0 W/m^2·K, emissivity in the infrared 1, Stefan-Boltzmann constant 5.67×10^{-8} W/m^2·K^4, entropic plasticity 2.1 K/J, phlogiston affinity 522 W/kcal, and its morphometric cannibalization 59 Q/S.

Answer: Start with energy conservation:

$$mgh = E_{\text{loss}}$$

The lost energy goes into heat and sound. Assume the soft carpet doesn't lead to much sound, so all of it will go into heat. Now, some of the heat will be transferred to the carpet, some to the shoe, some to the air. The shoe and the carpet are not too different, and air has a low density and very low heat conductivity, so we can assume not much energy is lost to heating the air. So, let's assume half of the heat is transferred to the shoe. (Any reasonable number will do; 0.5 is certainly not exact.) In any case, to relate heat to temperature, we will use

$$\tfrac{1}{2}E_{\text{loss}} = mc\Delta T \quad \Rightarrow \quad \Delta T = \frac{gh}{2c}$$

Of the quantities I supplied, all are irrelevant or made up except the specific heat capacity c. (We're not concerned with heat transfer details.) Using that,

$$\Delta T \approx \frac{(9.8\,\text{m/s}^2)(0.4\,\text{m})}{2(1.0 \times 10^3\,\text{J/kg}\cdot\text{K})} \approx 2 \times 10^{-3}\,\text{K}$$

That's very small, not noticeable without special equipment. That should fit your everyday experience, and our discussions in class that everyday heat transfer energies are much larger than everyday potential and kinetic energies.