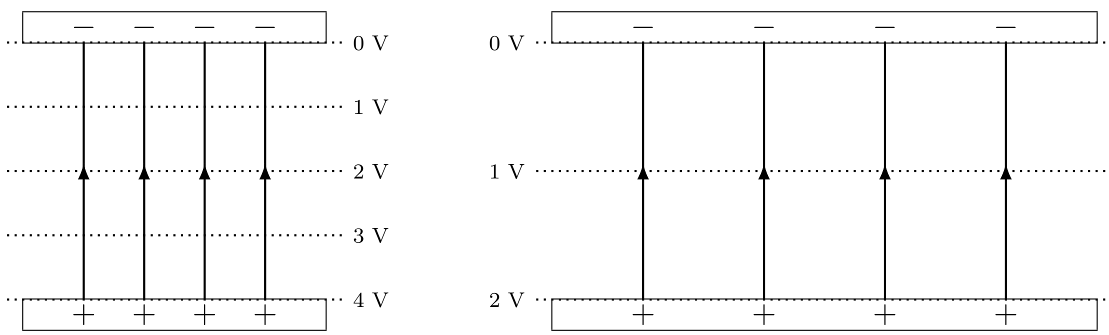
Solutions to Assignment 4; Phys 186

1. (20 points) Use the rules concerning equipotential lines and electric field lines to figure out what happens when you double the plate area of a parallel plate capacitor. You have a capacitor with plate area A and plate separation d with charges $\pm Q$ on its plates, with a voltage difference of 4.0 V between the plates. You also have a capacitor that is identical in every respect, except that the same charge is distributed throughout double the area, $2A$. What will the voltage reading on the second capacitor be? Draw electric field lines and equipotential lines at 1 V intervals, and explain your reasoning.

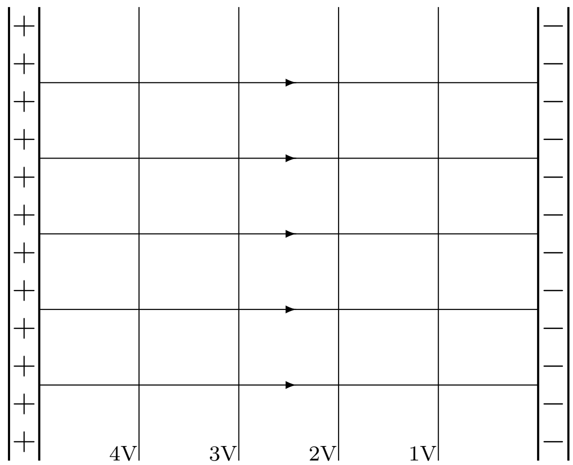
Remember that the the magnitude of the uniform electric field produced by an infinite plane of charge is proportional to the charge density (charge per unit area).



Answer: If the area is doubled while the amount of charge remains the same, the density of electric field lines will be halved. Since the electric field strength is proportional to the line density, this means the electric field strength will be halved. You can indicate this by drawing the electric field lines further apart. But if the lines are twice as far apart, that means the equipotential lines are spaced father apart, to give a more gentle slope of descent. In fact, they have to spaced twice as far apart, so the voltage can only go from 0 V to 2 V over the plate separation distance.

2. (30 points)

(a) You have a parallel plate capacitor. The left plate is set at a potential of 5 V, the right at 0 V. Draw in equipotential lines in between at 1, 2, 3, and 4 V, and the electric field lines. Finally, find an expression for the *total electrical force* by which the plates are attracted toward one another, in terms of the constant ϵ_0, the charge Q on each plate, the plate area A, and the plate separation d.



Answer: Each charge q on the plate will be in the constant electric field due to the other plate, with magnitude $E = Q/(2\epsilon_0 A)$—half of the full field. Therefore it will feel a force of magnitude $F = qQ/(2\epsilon_0 A)$. The total force is, since each charge feels the same force,

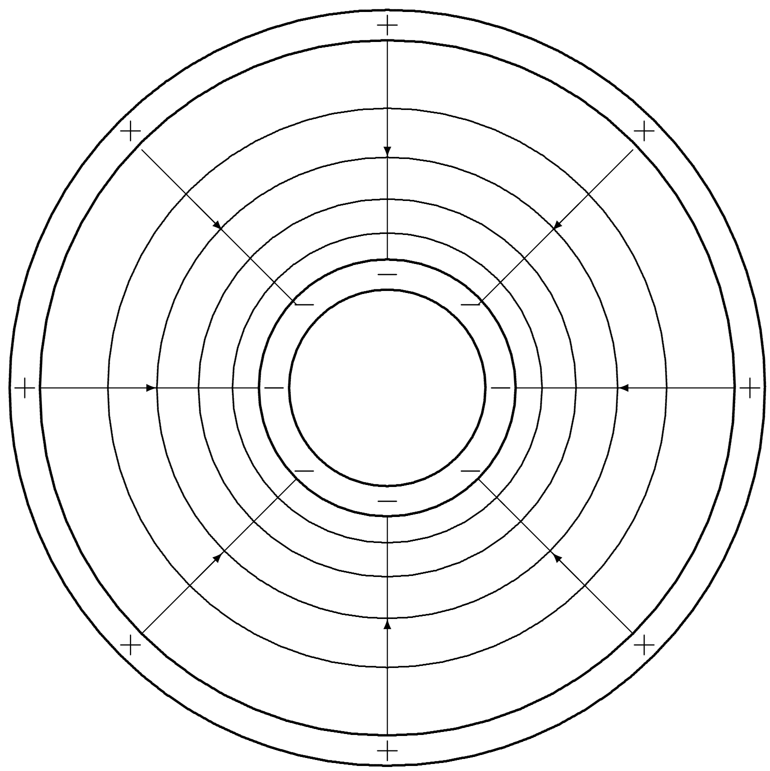
$$\sum F = \left(\sum q\right) \frac{Q}{2\epsilon_0 A}$$

But the total charge on a plate is $\sum q = Q$. Therefore

$$\sum F = \frac{Q^2}{2\epsilon_0 A}$$

and is attractive. (The plates are attracted to one another.)

(b) You now have a cylindrical capacitor: two concentric metal rings with equal and opposite charges. Draw a qualitative map of the equipotential lines and electric field lines for this case. Also include the voltage and electric fields inside the inner ring. What is the total electrical force acting on each ring?



Answer: From the symmetry, the electric field lines will be radial in direction, from the positively charged outer ring toward the negatively charged inner ring. Since the electric field lines will not be parallel, but will be getting closer together as they approach the inner ring, that means the electric field magnitude must be getting larger as you approach the inner ring. From all this, we can also figure out the equipotential lines. They are perpendicular to the electric field lines, which means they have to be concentric circles. And since the electric field magnitude becomes larger when the equipotential lines are close to one another, these circles must be increasingly close to one another as you get closer to the inner ring.

The voltage is constant inside the inner ring, and the electric field is zero. The total force is zero as well, from the symmetry of the situation: any charge on the inner ring that is attracted to the outer ring has a counterpart on the other side of the circle that feels an equal force in the opposite direction.

3. (25 points) You have two concentric metal spheres; the inner sphere has a total charge of $-Q$ while the outer sphere has $+2Q$. Draw the electric field lines and equipotential lines all over space: inside the inner sphere, in between the spheres, and outside the outer sphere. To determine the electric field strength, use

$$\text{(Electric flux through a closed surface)} = \frac{1}{\epsilon_0}\text{(Total charge inside that surface)}$$

$$\sum E_\perp A = \frac{Q_{\text{in}}}{\epsilon_0}$$

$$\int_{\partial S} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

These equations are all the same thing, in increasing order of mathematical propriety. What you care about is that if $\vec{E}$ is always constant and perpendicular to a closed surface,

$$EA = \frac{Q_{\text{in}}}{\epsilon_0}$$

where E is the outward electric field strength at the enclosing surface you choose, and A is the area of the surface. (I suggest figuring out E before drawing the lines. Check with me to see how you're doing.)

Answer: Since this is a spherically symmetric situation, your enclosing surfaces should be spheres, concentric with the charged spheres. The electric field will then always be perpendicular to the spherical surface, and always have a constant magnitude at the surface. Therefore your electric flux will be $EA = E(4\pi r^2)$, where r is the radius of your enclosing sphere. There are three regions:

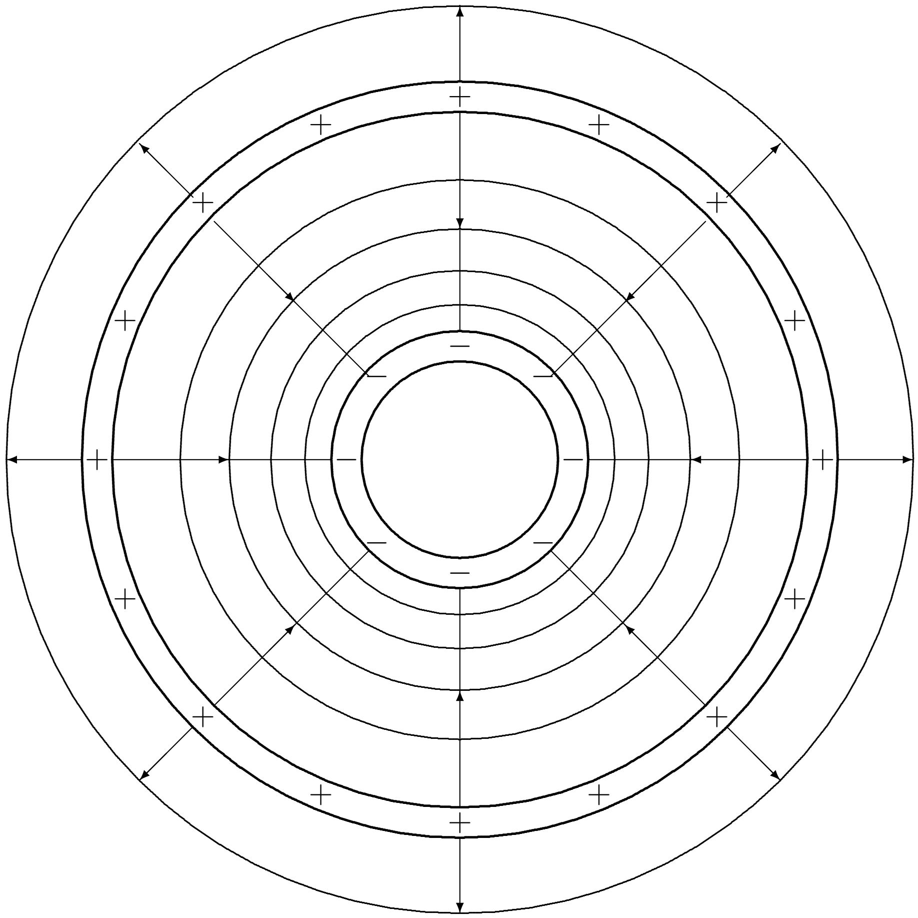
Inside the small sphere: In this case, $Q_{\text{in}} = 0$, therefore $E = 0$.

Between the charged spheres: $Q_{\text{in}} = -Q$, therefore $E(4\pi r^2) = -Q/\epsilon_0$ and

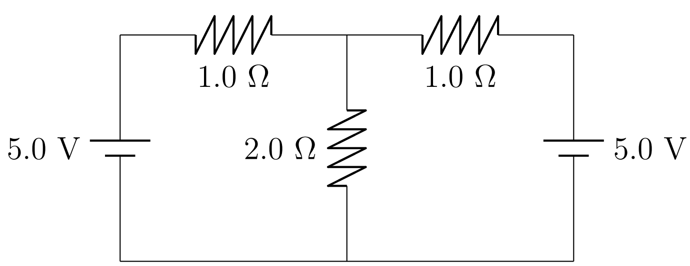
$$E = -\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = -k\,\frac{Q}{r^2} \qquad \text{(The } - \text{ sign means } \vec{E} \text{ points inward.)}$$

Outside both spheres: $Q_{\text{in}} = -Q + 2Q = Q$, therefore $E(4\pi r^2) = Q/\epsilon_0$ and

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = k\,\frac{Q}{r^2} \qquad \text{(The } + \text{ sign means } \vec{E} \text{ points outward.)}$$



4. (25 points) You have the following circuit. Calculate the voltage across, the current through, and the power dissipated by each resistor.



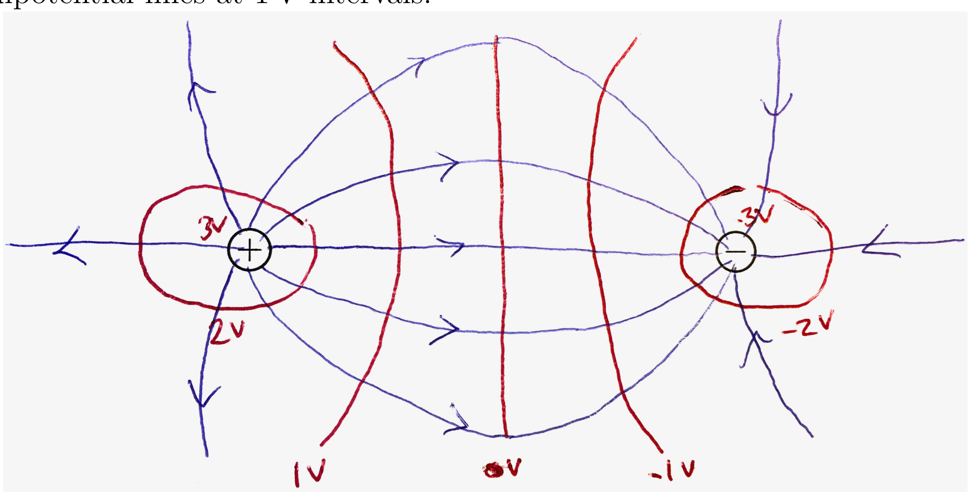
Answer: Use loops and junctions. Or use this shortcut: the left and right halves of this circuit are identical. So clearly the currents through the 1 Ω resistors are identical: call them I_1. Call the current through the

2 Ω resistance I_2. The junction equation then becomes $I_2 = 2I_1$. Both loop equations are identical: $5\text{ V} = (1\,\Omega)I_1 + (2\,\Omega)I_2$. Putting the equations together, we get $5\text{ V} = (1\,\Omega)I_1 + (2\,\Omega)2I_1$, which means $I_1 = 1$ A. Therefore $I_2 = 2$A.

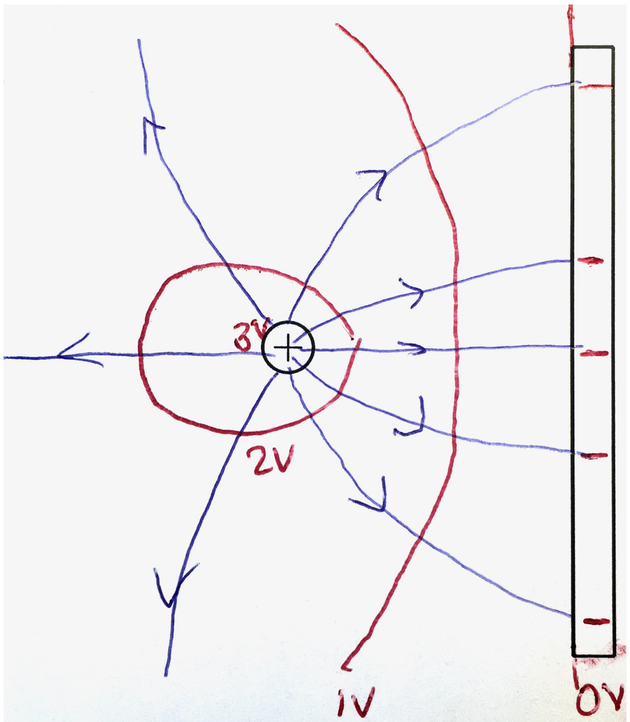
The voltages: $V_1 = (1\,\Omega)I_1 = 1$ V; $V_2 = (2\,\Omega)I_2 = 4$ V. The powers: $P_1 = V_1 I_1 = 1$ W; $P_2 = V_2 I_2 = 8$ W.

Extra Problems (not graded)

5. (0 points) First, draw equipotential lines and electric field lines for a dipole-like arrangement you investigated in the lab. The low-voltage point in the water is at -3 V, and the high-voltage point is at $+3$ V. Draw the equipotential lines at 1 V intervals.



Then, draw equipotential lines at 1 V intervals and electric field lines for the case where you have a point at $+3$ V and a plate at 0 V. Only draw what is happening on the left side; you can ignore the right side of the plate. Also draw some appropriate charges on the plate to give a qualitative idea of the charge distribution on the plate. Explain your reasoning.

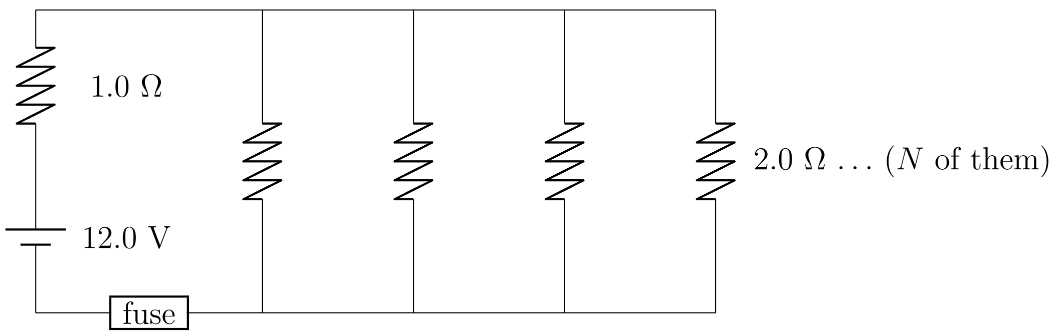


How are your two graphs related—how did drawing the dipole first help you draw the second graph with the plate?

Answer: The conducting plate is itself an equipotential. You will notice that the given equipotential lines at 3 V and 0 V are the same for the dipole and the charge-and-plate arrangements. Indeed, *everything* on the left side will be the same for the two arrangements, because the given voltage constraints are identical.

The charges on the plate will be negative—it's the low voltage end. They won't be distributed equally throughout the plate, since they are attracted toward the + charge.

6. (0 points) You have a circuit with a fuse in it to limit the current drawn from the battery, which supplies power to N 2.0 Ω resistors hooked in parallel. The 1.0 Ω resistor represents the internal resistance of the battery. The fuse acts like an ideal wire until a maximum current goes through it, at which point it burns up and breaks the circuit.



(a) Write down the junction, loop, and resistor equations for the circuit. Note that this is not as complicated as it looks, since many of your equations will be simplified to become identical. Ask me what I mean if you're confused.

Answer: Call the current from the battery I_0 and the currents through the resistors I_1 through I_N. There is a single junction equation, when the battery current splits into N parts:

$$I_0 = I_1 + I_2 + \ldots + I_N$$

The other junction gives the same equation, since all the split currents rejoin.

The resistor equations are $V = RI$ for each; it's easiest to directly use these in the loop equations. There are N independent loops in the circuit. The first loop equation is

$$12 \text{ V} = (1\,\Omega)I_0 + (2\,\Omega)I_1$$

The $N-1$ other loop equations are

$$(2\,\Omega)I_1 = (2\,\Omega)I_2, \quad (2\,\Omega)I_2 = (2\,\Omega)I_3, \quad \ldots \quad (2\,\Omega)I_{N-1} = (2\,\Omega)I_N$$

All this means is that the split currents are all equal:

$$I_1 = I_2 = \ldots = I_N$$

So we end up with two equations

$$I_0 = N\,I_1, \qquad 12 \text{ A} = I_0 + 2\,I_1$$

and three unknowns, I_0, I_1, and N.

(b) If the fuse blows when the current through it is more than 10.0 A, what is the maximum number N_{max} of 2.0 Ω resistors that can be hooked up in this circuit?

Answer: We just need to solve our equations for when $I_0 = 10$ A. In that case, $I_1 = 10/N_{\text{max}}$ A, and we are left with

$$12 \text{ A} = 10 \text{ A} + 2 \frac{10}{N_{\text{max}}} \text{ A}$$

which means that $N_{\text{max}} = 10$.

(c) Could you increase N_{max} by hooking up a capacitor in series with the 1.0 Ω resistor? In parallel? Explain.

Answer: If you hooked up a capacitor in series, it would charge up, and once it was fully charged, no more charges would be moving on that wire: the current I_0 would become zero. That would not help.

In parallel, again, the capacitor would charge up, and no current would go through afterwards. But that would just leave us with an equivalent of the original circuit after a while, so N_{max} would not be affected.

In other words, a capacitor would not do the job, however it is connected.

(d) What does this problem tell you about the consequences of hooking up lots of appliances to a single wall socket by using extension cords?

Answer: If you keep adding appliances, your fuse or circuit breaker will have to break the circuit so you don't draw too much current and start a fire.