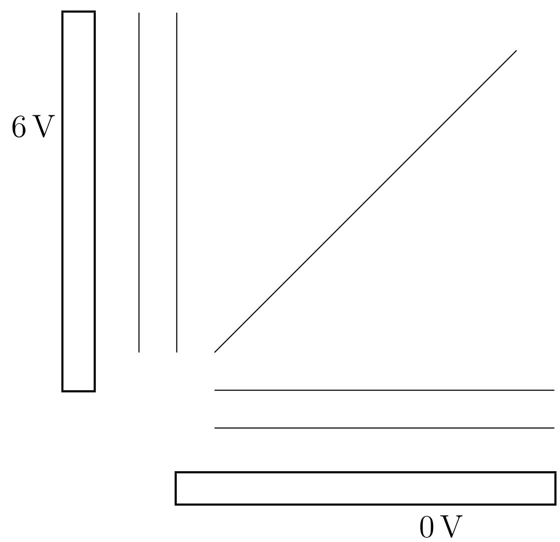
Solutions to Exam 2; Phys 186

1. (20 points) In your equipotential lines lab, you set up two metal plates at right angles. You set the voltage of the plate drawn vertically here to 6 V, and the voltage of the horizontal one to 0 V. You then step outside the lab for ten minutes, so your lab partner maps equipotential lines at 1, 2, 3, 4, and 5 V. Here is the map you see when you return.



(a) Your partner has made a mistake! You should notice that when you draw in the electric field lines and pay attention to their spacing. Draw, below, what the equipotential lines should have looked like, and the correct electric field lines.

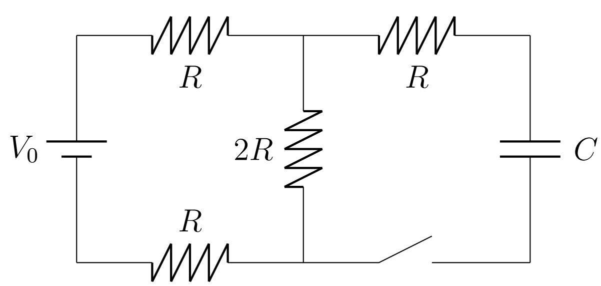
Answer: The mistake is that the equipotential lines other than the one in the middle are too close and too parallel to the plates. It's hard to draw electric field lines perpendicular to these and to also space them further apart when you get to the space in the middle.

Draw new equipotential lines that fan out from where the plates approach one another. The electric field line will be quarter-circle curves between the plates, and their spacing will increase away from the center.

(b) Draw + and − charges inside the metals to indicate the charge distribution.

Answer: There will be + charges on the high voltage plate and − charges on the low voltage plate. These charges will attract one another, being concentrated closer together where the plates get close to one another. This is also consistent with the elctric field being stronger in the region where the plates are close.

2. (40 points) You have the following circuit, with V_0, R, and C being known values. The capacitor starts out uncharged, and at time $t = 0$, you close the switch.



(a) Write down the junction and loop equations for this circuit for $t \geq 0$. Clearly draw, label and indicate all quantities you introduce on the circuit diagram.

Answer: Call the battery current I_0, the current through the $2R$ resistor I_2, and the current through the capacitor I_c. The junction equation is then

$$I_0 = I_2 + I_c$$

The two loop equations are, with $V = RI$ used for the resistors,

$$V_0 = RI_0 + (2R)I_2 + RI_0 \qquad \text{and} \qquad (2R)I_2 = RI_c + V_c$$

You can also leave this in the form of just voltages, as long as your labeling is clear.

(b) Find the current through and voltage across the $2R$ resistor at $t = 0$ (immediately after the switch is closed) and for $t \gg RC$.

Answer: At $t = 0$, $V_c = 0$. The second loop equation then gives $I_c = 2I_2$. Putting that into the junction equation, $I_0 = 3I_2$. Finally,

$$V_0 = 6RI_2 + 2RI_2 \qquad \Rightarrow \qquad I_2 = \frac{V_0}{8R}$$

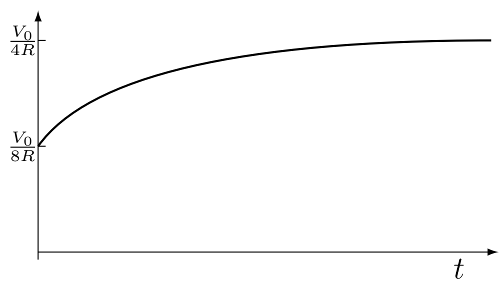
The voltage across is $V_2 = 2RI_2 = V_0/4$.

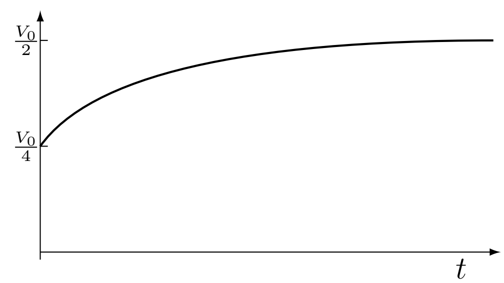
When $t \gg RC$, $I_c = 0$. The junction equation becomes $I_0 = I_2$, and the second loop equation gives

$$V_0 = 2RI_2 + 2RI_2 \qquad \Rightarrow \qquad I_2 = \frac{V_0}{4R}$$

The voltage across is $V_2 = 2RI_2 = V_0/2$.

(c) Sketch rough graphs for the current through and voltage across the $2R$ resistor, and explain why you chose the shapes of the graphs as you did.

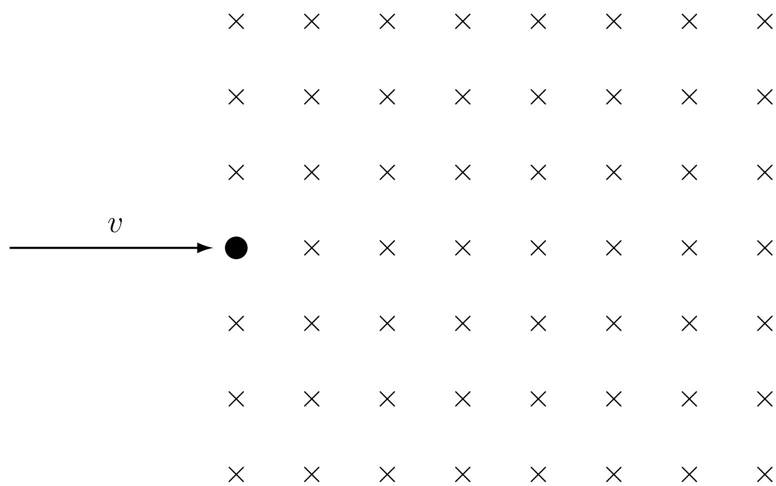




Answer: This circuit is very similar to your Assignment 5 question 2; the explanation is the same.

3. (40 points) A charge q ($q > 0$) accelerated through a voltage difference of V_a, moving toward the right, enters a region with a uniform magnetic field with magnitude B. The magnetic field points into the page.

Draw the trajectory of the charge within the region with the uniform magnetic field. You will notice that the charge exits the region; you measure the distance d between the point of entry of the charge and the point of exit. Find an equation for m, the mass of the charge. *Note:* Check with me once you have a result. Only known quantities q, V_a, B, and d should appear in your equation.



Answer: The charge will do circular motion, tracing a half-circle before exiting back in the direction it came from. Therefore, $d = 2r$, where r is the radius of the circle.

The speed of the charge is, following your Assignment 6 question 1, found through energy conservation:

$$\tfrac{1}{2}mv^2 = qV_a \qquad \Rightarrow \qquad v = \sqrt{\frac{2qV_a}{m}}$$

The magnetic force is the centripetal force, so

$$\frac{mv^2}{r} = qBv \qquad \Rightarrow \qquad mv = qBr = qB\frac{d}{2}$$

Doing the algebra,

$$\left(m\sqrt{\frac{2qV_a}{m}}\right)^2 = \left(qB\frac{d}{2}\right)^2 \qquad \Rightarrow \qquad m = \frac{qB^2d^2}{8V_a}$$

This, by the way, is how a *mass spectrometer* works.