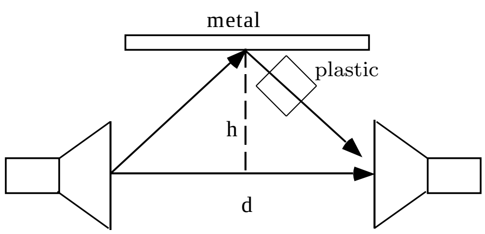
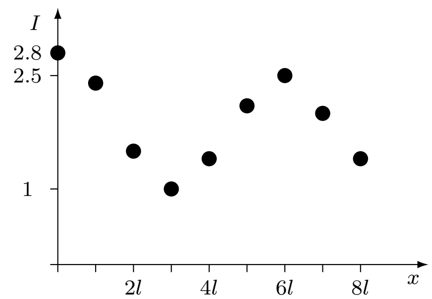
Solutions to Exam 1; Phys 186

1. (60 points) You do an experiment similar to Part 2 of Lab 3, where you reflect microwaves off a metallic surface as well as having the microwaves travel directly from source to detector. You use more expensive equipment to measure the distances d and h, the wavelength λ, and the intensity I, more precisely.

You start your experiment at a point where I is a maximum—you have constructive interference. Then, you insert a small slab of a plastic material with thickness l into the path of the microwaves bouncing off the metal. You record the intensity I at various thicknesses of the same plastic material, from 0 to $8l$, getting a graph that looks like below.





(a) You have two explanations proposed for this graph. Choose the correct one.

(i) The index of refraction of the plastic material is $n_{\text{pl}} > 1$, while $n_{\text{air}} = 1$. Inserting the plastic produces a phase shift between the waves in the two paths from source to detector. If this is correct, find an equation for n_{pl}, using the intensity graph and quantities known in the experiment. *Hint:* how many wavelengths fit into a thickness of plastic x? How many wavelengths would have fit in x if f the plastic wasn't there?

(ii) The plastic is mostly transparent to microwaves, but it absorbs some of the wave energy. The intensity behaves as $I = I_0 e^{-x/\alpha}$, where x is the thickness of plastic and α is known as the attenuation length. If this is correct, determine α, using the intensity graph and quantities known in the experiment. *Hint:* When

$x = \alpha$, I will fall to I_0/e, 37% of the original value. You should be able to estimate this $x = \alpha$ from the graph.

Answer: The first option is correct: the intensity graph is obviously not an exponential decay.

Without plastic, the number of wavelengths that fit into a distance x is x/λ. But the wavelength changes in plastic! Since $n = c/v = c/\lambda f$, and since f and c are constants, the wavelength inside the plastic is λ/n_{pl}. The number of wavelengths that actually fit into a plastic thickness of x is $n_{\mathrm{pl}}x/\lambda$. So the path length difference between the two paths from source to detector is not what it was without plastic: there is a phase shift of $(n_{\mathrm{pl}}x/\lambda - x/\lambda)\lambda = (n_{\mathrm{pl}} - 1)x$. When this phase shift is $\lambda/2$, we get destructive interference, a minimum. When this phase shift is a full wavelength, we should get another intensity maximum, as constructive interference will be restored. The minimum happens at $x = 3l$, the maximum at $x = 6l$. Therefore $(n_{\mathrm{pl}} - 1)6l = \lambda$ and

$$n_{\mathrm{pl}} = 1 + \frac{\lambda}{6l}$$

This is a a good method to measure indices of refraction, by the way.

(b) The second intensity peak in the graph, at plastic thickness $6l$, is lower than the first intensity peak with no plastic. What is the explanation for this?

Answer: This is because the plastic does actually absorb some of the energy transmitted by the microwaves, so the intensity drops a bit.

(c) The microwaves reflecting from the metal are inverted, picking up an extra phase shift of $\lambda/2$. Do you need to take this into account in your previous answers? Explain.

Answer: It's irrelevant. The intensity graph captures the fact that the intensity maxima are separated by a plastic thickness of $6l$, and that's all that matters.

2. (40 points) You have four charges arranged in a line. All have $x = 0$, and $y = \pm a$ for charges $\pm q$ and $y = \pm 2a$ for charges $\pm Q$.

(a) Draw arrows indicating the electric fields produced by each charge at an arbitrary point on the x-axis.

(b) Find the total electric field at a point a distance x from the origin on the x axis. In other words, find the total E_x and E_y. *Hint:* You don't need to calculate this from scratch. Just use results known to you from your notes or your assignments.

Answer: There are two dipoles here: one with $\pm q$ and separation $2a$, the other with $\mp Q$ and separation $4a$ (replace q with $-Q$ and a with $2a$). Therefore,

$$E_{\text{total},x} = 0 - 0 = 0$$

$$E_{\text{total},y} = -\frac{2kqa}{(x^2 + a^2)^{3/2}} + \frac{2kQ(2a)}{(x^2 + (2a)^2)^{3/2}}$$

(c) Find the approximate total dipole electric field when $x \gg a$.

Answer: If $x \gg a$, then $x^2 + a^2 \approx x^2$ and $x^2 + 4a^2 \approx x^2$. Therefore

$$E_{\text{total},y} \approx -\frac{2kqa}{x^3} + \frac{4kQa}{x^3} = \frac{2kqa}{x^3}(2Q - q)$$

(d) What must Q be for the dipole contribution to the electric field to be equal to zero? (Look at your answer for $x \gg a$.)

Answer: $Q = q/2$ would produce a zero result.

(e) If Q is set to make the dipole contribution to the electric field zero, does this mean that the actual electric field when $x \gg a$ is exactly zero?

Answer: No; your answer to (b), which is not approximated, will not be exactly zero. (It will be very small, though: it will only have quadrupole and higher pole contributions.)