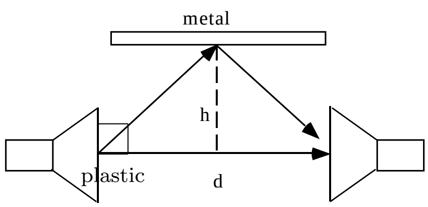
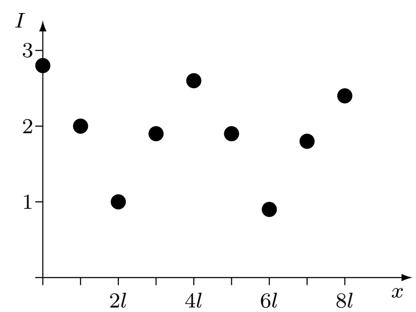
Solutions to Exam 4; Phys 186

1. (30 points) You do an experiment similar to Part 2 of Lab 3, where you reflect microwaves off a metallic surface as well as having the microwaves travel directly from source to detector. You have $d = 30.0\,\mathrm{cm}$ and the wavelength $\lambda = 3.00\,\mathrm{cm}$. You start your experiment at a value of h where the intensity I is a maximum—you have constructive interference. Then, you insert a small slab of a plastic material with index of refraction $n_{\mathrm{pl}} = 1.75$ and thickness $l = 1.00\,\mathrm{cm}$ into the *straight* path for the microwaves. You record the intensity I at various thicknesses of the same plastic material, from 0 to $8l$. I has arbitrary units, just as in Lab 3. Plot what you think the data points from such an experiment would look like. Provide calculations and explanations below.





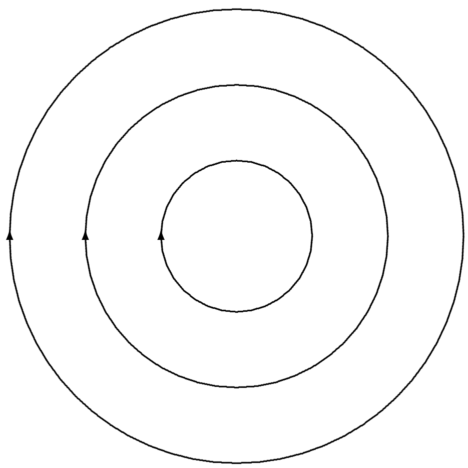
Answer: In plastic, $\lambda_{\mathrm{pl}} = \lambda/n_{\mathrm{pl}}$, therefore, the extra phase shift due to a thickness ml of plastic is $s_m = (n_{\mathrm{pl}} - 1)ml$. Therefore

$$s_m = m(0.75\,\mathrm{cm}) = \tfrac{m}{4}\lambda, \quad m = 1, 2, \ldots, 8$$

This means maxima at $m = 0, 4, 8$ with integer numbers of wavelengths as the extra phase shift, and minima at $m = 2, 6$ with half-integer numbers of wavelengths. I've drawn I as declining somewhat with x to allow for some absorption in the plastic.

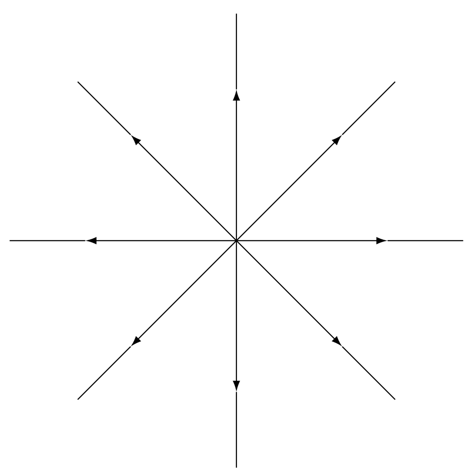
2. (15 points) Consult the four qualitative "Maxwell's equations" I described, concerning how electric and magnetic fields radiate and circulate, to answer the following:

(a) In the lab, you want to produce an *electric field* that looks like below. Is this possible? If not, explain why. If yes, what would you need in the lab to make this happen? (In broad terms; no need for details about equipment.)



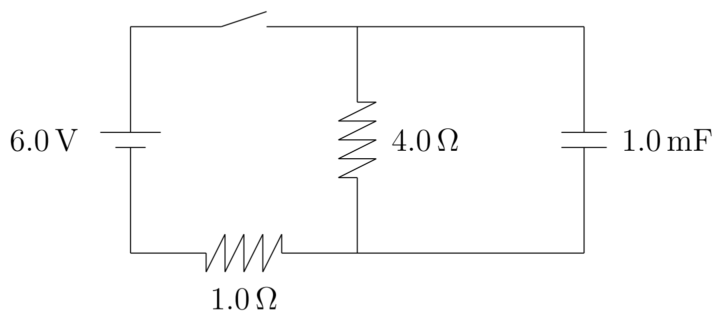
Answer: A circulating electric field is produced not by charges but by a changing magnetic flux. So we need a setup with a changing magnetic flux.

(b) In the lab, you want to produce a *magnetic field* that looks like below. Is this possible? If not, explain why. If yes, what would you need in the lab to make this happen? (In broad terms; no need for details about equipment.)



Answer: A radiating magnetic field would be produced by magnetic charges (monopoles), but as far as we know, such things do not exist, so you would not find this to be possible in the lab.

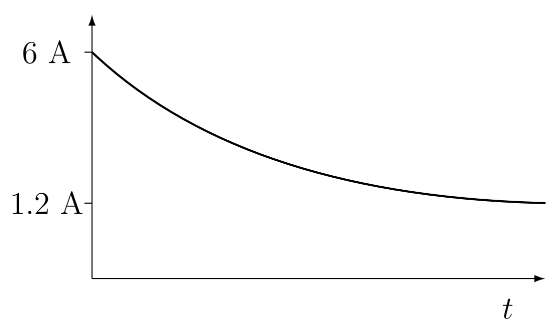
3. (20 points) You keep the switch open for 60.0 seconds. Then, at time $t = 0$, you close the switch. Sketch a *qualitative* graph of the current through the $1.0\,\Omega$ resistor versus time t. On your graph, indicate the exact numerical value for this current at $t = 0$, immediately after the switch is closed, and the value of this current as $t \to \infty$, after a very long time.



Answer: At $t = 0$, the capacitor starts charging up, and short-circuits the $4\,\Omega$ resistor. We get $I_1 = (6\,\text{V})/(1\,\Omega) = 6\,\text{A}$.

As $t \to \infty$, the capacitor will be fully charged, and we can take it out of the circuit. We will be left with the two resistances in series. The current will be $I_1 = (6\,\text{V})/(1\,\Omega + 4\,\Omega) = 1.2\,\text{A}$.

In between, the current will decay in a characteristic exponential fashion, but not down to zero.



4. (20 points) Recall the calculation where we derived the radius of the event horizon of a non-rotating black hole, $r = 2Gm/c^2$. This used energy conservation to get the escape velocity from a spherical object.

(a) Is the kinetic energy equation we used in this calculation ($K = \frac{1}{2}mv^2$) (i) exactly correct, (ii) a good approximation for the case of black holes, or (iii) a very dubious approximation for the case of black holes? Explain why.

Answer: This is a very dubious approximation. $K = \frac{1}{2}mv^2$ is valid only when $v \ll c$, but setting the escape velocity $v = c$ is exactly what we need to get the black hole result.

(b) Is the gravitational potential energy equation in this calculation ($U = -GmM/r$) (i) exactly correct, (ii) a good approximation for the case of black holes, or (iii) a very dubious approximation for the case of black holes? Explain why.

Answer: This is a very dubious approximation. Newtonian gravity, where $U = -GmM/r$, is valid only for weak gravity. But black holes involve extremely strong gravity.

(c) The equation $r = 2Gm/c^2$ is exactly correct according to general relativity (it's not an approximation). Do you think it's because (i) our calculation in class was exact, or (ii) there's a lucky cancellation of the error in the kinetic energy and the error in the potential energy, that I didn't tell you about? Explain.

Answer: Since $r = 2Gm/c^2$ is exact, the errors introduced by the two dubious approximations must have cancelled out. Indeed, that's exactly what happens.

5. (15 points) According to non-quantum physics, the H atom should not exist.

(a) Briefly explain why H should not exist. Emphasize fundamental physical principles in your explanation.

Answer: In non-quantum H, the electron must be in orbit around the much heavier proton. Doing circular motion, the electron therefore accelerates. Accelerating charges broadcast electromagnetic radiation. Energy is conserved, therefore the energy carried off by the electromagnetic waves must come from the H atom. We have $E = \frac{1}{2}mv^2 - ke^2/r$ as the total energy, and $\sum \vec{F} = m\vec{a}$ gives $ke^2/r^2 = mv^2/r$. Combining these,

$$E = -k\frac{e^2}{2r}$$

Losing energy must mean that r becomes smaller. Electromagnetism is a strong force, therefore the electron will spiral down toward the proton *quickly,* and no H will exist.

(b) Briefly explain how quantum physics allows H to exist. Emphasize the uncertainty principle in your explanation.

Answer: Due to $\Delta p \Delta x \geq \hbar/2$, we should think of the electron not as having a definite location and momentum, but a probability distribution with finite width for each. The electron in the ground state of H has a probability distribution such that its highest likelihood point is on the proton, but otherwise the distribution is spread out. The electron also does not revolve around the proton, hence it does not radiate and lose energy.