Solutions 2; Phys 191 Name ________________

A *sinusoidal traveling wave* has the form

$$y(x,t) = A\cos(kx - \omega t - \delta)$$

Notice its similarity to the harmonic oscillator.

1. (20 points) This is a traveling wave, so it should equal $F(x - vt)$ for some F and v. What is the speed of this wave (in terms of A, k, ω, δ)?

Answer: Rewrite the equation as

$$A\cos(kx - \omega t - \delta) = A\cos\left[k\left(x - \frac{\omega}{k}t\right) - \delta\right]$$

This is a function of $x - vt$ with $v = \omega/k$.

2. (20 points) As in your last assignment, interpret the physical meaning of A, k, ω, and δ, and find relations between these variables and the period T and wavelength λ of the wave.

Answer: A is the amplitude of the wave, $k = 2\pi/\lambda$ the wave number, $\omega = 2\pi/T$ the angular frequency, and δ a phase shift.

3. (60 points) Waves can travel in both directions along a string simultaneously. Suppose we have two sinusoidal traveling waves:

$$y(x,t) = A\cos(kx - \omega t) + B\cos(-kx - \omega t)$$

These waves have the same frequency but different directions and possibly different amplitudes.

Now suppose we clamp our string at two points: $x = 0$ and $x = L$. This means that $y = 0$ at those two points at *all* times t. Use that condition to eliminate variables: you should be able to find B in terms of A, and k in terms of L. You should note that there are multiple solutions for k—find the condition for allowed k values. A helpful trigonometric identity: $\cos(A + B) = \cos A\cos B - \sin A\sin B$.

You should find that only special values of k (and therefore ω) are allowed, but the amplitude is not restricted. These waves are called *standing waves* (because they have no overall translational motion) and the special frequencies are *resonant* or *harmonic* frequencies.

Answer: Impose the boundary conditions at $x = 0$ and $x = L$:

$$y(0,t) = A\cos(\omega t) + B\cos(\omega t) = 0 \qquad \Rightarrow \qquad A = -B$$

$$y(L,t) = A\cos(kL - \omega t) - A\cos(-kL - \omega t) = 0 \quad \Rightarrow \quad kL = n\pi,\ n = 1, 2, 3, \ldots$$

The reason that $kL = n\pi$ is that the cosine function has a periodicity of 2π, therefore

$$\cos(\phi_1 - \omega t) = \cos(\phi_2 - \omega t) \quad \Rightarrow \quad \phi_1 - \phi_2 = m2\pi,\ m = 0, \pm 1, \pm 2, \ldots$$

In our case, $\phi_1 - \phi_2 = 2kL$. But the $m = 0$ case means that $k = 0$ and therefore $\lambda = \infty$; that's a constant, and you can only satisfy the boundary conditions when $A = 0$. That's no wave at all. And since $\cos\theta = \cos(-\theta)$, the negative m solutions are the same as the corresponding positive m solutions. So for actual standing waves, we end up with discrete k's restricted to the values

$$k = \frac{n\pi}{L}, \quad n = 1, 2, 3, \ldots$$