Solutions to Exam 1; Phys 185

1. (50 points) Lab 3, where you figured out g, depended on your letting the cart go very close to the first light gate, so that its initial velocity was as close to zero as possible. That was hard to do, which introduced a major source of error into your experiment.

Say you decide to fix the problem by letting the cart go a distance d before the first gate. Your final equation for g will now be different: it will incorporate d. Find this final equation.

Note: You don't need to reproduce the full calculation from the pre-lab and what we did in class. Much of it won't be affected by letting the cart go at d. However, do explain your reasoning: if you re-use a result without changing it, give me a sentence explaining why.

Answer: The forces, and therefore the acceleration of the cart, are not affected by starting at d. Therefore the equation we obtained in class (and in the pre-Lab)

$$g = \frac{m_c + m_h}{m_h}\, a$$

does not change. What changes is that we can no longer use $a = 2\Delta x/(\Delta t)^2$.

You still record $\Delta t = t_2 - t_1$, the difference between the times of the cart's entry into light gate 1 and 2. Using the equations for motion with constant acceleration, starting from rest at $t = 0$, we get

$$d = \tfrac{1}{2} a t_1^2 \quad \text{and} \quad d + \Delta x = \tfrac{1}{2} a t_2^2$$

Therefore,

$$\Delta t = t_2 - t_1 = \sqrt{\frac{2}{a}(d + \Delta x)} - \sqrt{\frac{2}{a} d}$$

Solving for the acceleration

$$a = 2 \left(\frac{\sqrt{d + \Delta x} - \sqrt{d}}{\Delta t} \right)^2$$

Notice that when $d = 0$, this becomes the acceleration equation we have used. Now, just put that into the equation for g:

$$g = 2 \left(\frac{m_c + m_h}{m_h} \right) \left(\frac{\sqrt{d + \Delta x} - \sqrt{d}}{\Delta t} \right)^2$$

2. (50 points) You're moving a box with mass m on a horizontal surface, at constant velocity with $v > 0$. There is friction, with coefficients μ_s and μ_k, but your speed is not large enough to take drag into consideration. You have two options: (i) pull with a force directed at an angle θ *above* the horizontal, or (ii) push the box downwards at the same angle θ below the horizontal.

(a) Find equations for the force magnitude F required in both cases (i) and (ii). In which case will you need a larger force?

Answer: Constant velocity means the acceleration is zero: $a_x = a_y = 0$. Since $v > 0$ we have kinetic, not static, friction.

Case (i):

$$\sum F_x = F\cos\theta - f_k = F\cos\theta - \mu_k n = 0$$
$$\sum F_y = F\sin\theta + n - w = F\sin\theta + n - mg = 0$$

From the second equation, $n = mg - F\sin\theta$. Putting that into the first equation,

$$F\cos\theta - \mu_k(mg - F\sin\theta) = 0 \quad \Rightarrow \quad F = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

Case (ii) is the same, except that F_y changes sign. Therefore

$$F = \frac{\mu_k mg}{\cos\theta - \mu_k \sin\theta}$$

With the minus sign, case (ii) involves division by a smaller number, so you need a larger force. That should make sense, from your everyday experience.

(b) Say you moved fast enough that you need to take the drag force into account. Does that change your answer about the force for (i) or (ii) being larger? Why?

Answer: Including the drag force means an extra $D_x = -D$. So

$$F = \frac{\mu_k mg + D}{\cos\theta \pm \mu_k \sin\theta}$$

That doesn't change what is larger.

(c) For both cases, write down your no-drag answers from part (a) for $\theta = 0$ and $\theta = 90°$. Interpret your $\theta = 90°$ results. What do they mean?

Answer: Case (i): $\theta = 0$ gives $F = \mu_k mg$, $\theta = 0$ gives $F = mg$. At right angles, the force is pointing straight up, and the only way it can negate friction is to make $n = 0$ by canceling the weight alone.

Case (ii): $\theta = 0$ gives $F = \mu_k mg$, $\theta = 0$ gives $F = -mg$. Pressing straight down means your force can no longer cancel out friction. Since F is a magnitude, $F \geq 0$, but the result is negative. This means that no F can result in a $v > 0$.

(d) In your results in (a) for F, for either case, is there an angle $0 \leq \theta \leq 90°$ for which you divide by zero, leading to an infinite F? If so, what does that mean?

Answer: Case (i) has the + sign, and there can be no division by zero. If you're pulling slightly upward, there is always a finite force which allows you to cancel out friction and move the box at constant velocity.

Case (ii) leads to division by zero when $\cos\theta = \mu_k \sin\theta$. For $\theta \geq \theta_{max} = \cot^{-1}\mu_k$, it is not possible to have $v > 0$ constant velocity motion any more. You already know that it's a bad idea to try to move a box while partly pushing down on it.