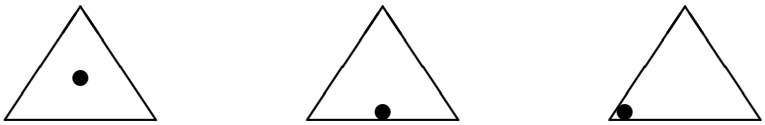
# Solutions to Exam 2; Phys 185

**1. (20 points)** You have an equilateral triangle cut out of a flat piece of metal with uniform density. You attach it to a motor that provides a constant torque, and apply the motor for an equal amount of time, starting from rest. You do this using three different axes of rotation. These axes are indicated as a black dot. Order the angular velocities of the three situations: which is the largest, which is the smallest, are any equal? Explain your reasoning.

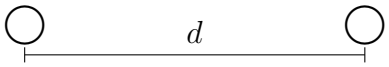


**Answer:** Since $\sum \tau = I\alpha$ and $\omega = \alpha t$, the triangle that spins the fastest will have the lowest moment of inertia $I$. The moment of inertia depends on the mass distribution, if more of the mass is distributed closer to the axis of rotation, $I$ will be smaller.

$I$ is smallest when the axis is at the center of the triangle (also the center of mass). That one will have the largest $\omega$. Next comes the one where the axis is at the bottom of the triangle. The largest $I$ and the smallest final $\omega$ happens when the axis is at a corner of the triangle, as the axis is then farthest from the center of mass.

You can use the parallel axis theorem, $I = I_{\text{cm}} + mr^2$, to more quantitatively express this argument, but pointing to the mass distribution will be good enough.

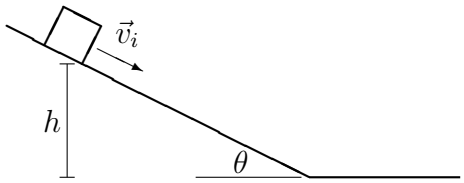
**2. (30 points)** You have two identical stars with mass $m$ revolving around each other in circular orbits. Everything else in space is too far to significantly affect them. The distance between the centers of the two stars is $d$. Find an equation for their period of rotation $T$. This equation can have (but need not have) the symbols $m$, $d$, $G$, $g$, $h$, $x$, $\tau$, and $k$ in it, as well as pure numbers.




**Answer:** The stars revolve around their common center of mass, which is a distance $r = d/2$ from them. For either star, $\sum \vec{F} = m\vec{a}$ gives

$$G\frac{m^2}{d^2} = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 m\frac{d}{2}}{T^2} \quad \Rightarrow \quad T = \pi\sqrt{\frac{2d^3}{Gm}}$$

**3. (20 points)** You have a cart on a ramp angled at $\theta$, starting out at a height $h$ from the level table and an initial downward speed of $v_i$. It never goes fast enough for drag to be important, but there is friction. The coefficient of kinetic friction between the cart and the ramp is $\mu_k$. Find $v_f$, the speed at which the cart will be going when it emerges from the ramp onto the flat table. *Reminder:* don't hesitate to ask me for trigonometry help!




**Answer:** The work done by friction is $W = -\mu_k n d$, where the distance $d = h/\sin\theta$ and $n = mg\cos\theta$. This means that

$$E_{\text{loss}} = -W = \mu_k mgh\cot\theta$$

Energy conservation then gives

$$\tfrac{1}{2}mv_i^2 + mgh = \tfrac{1}{2}mv_f^2 + \mu_k mgh\cot\theta \quad \Rightarrow \quad v_f = \sqrt{v_i^2 + 2gh(1-\mu_k\cot\theta)}$$

**4. (30 points)** Say you have a variable $f$ that depends on time, $f(t) = Af_1(t) + Bf_2(t)$, where $A$ and $B$ are constants. In that case, the rate of change of $f$ is

$$\frac{df}{dt} = A\frac{df_1}{dt} + B\frac{df_2}{dt}$$

Consider two carts with masses $m_1$ and $m_2$ on a level track, in a lab with expensive equipment that makes friction and drag negligible. The position of carts 1 and 2 are $x_1(t)$ and $x_2(t)$.

(a) The center of mass velocity of the carts is $v_{\text{cm}} = \frac{d}{dt}x_{\text{cm}}$. Find what $v_{\text{cm}}$ is in terms of $v_1$, $v_2$, $m_1$, and $m_2$. Check with me to make sure you get this right!

**Answer:** Using the definition of the center of mass,

$$v_{\text{cm}} = \frac{d}{dt}\left(\frac{m_1x_1 + m_2x_2}{m_1+m_2}\right) = \frac{m_1\frac{dx_1}{dt} + m_2\frac{dx_2}{dt}}{m_1+m_2} = \frac{m_1v_1 + m_2v_2}{m_1+m_2}$$

(b) The two carts collide. Which of the following is the relationship between $v_{\text{cm}i}$, before the collision, and $v_{\text{cm}f}$, after the collision? Show your work.

(i) $v_{\text{cm}f} = m_1m_2v_{\text{cm}i}/(m_1+m_2)$

(ii) $v_{\text{cm}f} = (m_1+m_2)v_{\text{cm}i}/(m_1m_2)$

(iii) $v_{\text{cm}f} = (m_1-m_2)v_{\text{cm}i}/(m_1+m_2)$

**(iv)** $v_{\text{cm}f} = v_{\text{cm}i}$

(v) $v_{\text{cm}f} = -v_{\text{cm}i}$

**Answer:** Since you don't know $E_{\text{loss}}$, you can't use energy conservation. But since $\sum \vec{F}_{\text{ext}} = 0$, momentum conservation is fine.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Divide both sides by the total mass:

$$\frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2} \quad \Rightarrow \quad v_{\text{cm}i} = v_{\text{cm}f}$$

(c) Say one of your carts incorporated a compressed spring that could be released upon the collision, adding extra energy to the carts. Would that change your answer? Explain.

**Answer:** No change. You didn't use energy conservation, and momentum conservation is not affected by the spring, as it provides an internal force.